

Annex 7

The Requirements of Basic Academic Attainments for Senior Secondary Mathematics

I. Basic rationale

Mathematics is a science that studies the space forms and quantity relations. It is the common cultural heritage of mankind and its contents, thinking, methods and languages are important components of human culture. Being a scientific language and an effective tool, mathematics has exerted inestimable influence on the process of human development. With the arrival of the information society, mathematical literacy has become the basic literacy for modern citizens and mathematics has been more widely used in different aspects of society and daily life. The senior secondary mathematics curriculum further develops the necessary basic mathematical knowledge and skills, application ability and innovative awareness, thus students can further improve their mathematical thinking skills and problem solving capabilities. On these grounds, the requirements of Basic Academic Attainments for Senior Secondary Mathematics shall comply with the following fundamental principles.

(1) The Mathematics curriculum should construct a common foundation.

The senior secondary mathematics curriculum should cover fundamental elements of mathematics, which includes significance in two aspects: first, it provides students with a higher mathematical foundation for adapting to modern life and future development; second, it provides students with the essential mathematical preparation for their further studies. The senior secondary mathematics curriculum aims to satisfy the common mathematical needs of all students and it is the common mathematical foundation for their development.

(2) Mathematics learning should be based on understanding, reflecting the diverse ways of learning.

Mathematics learning is a process of discovering new knowledge based on previous knowledge and experience. Effective mathematics learning activities cannot simply rely on memorisation and imitation; mathematical experiments, investigations and exchanges are also important modes of mathematics learning. The senior secondary mathematics

curriculum should advocate such learning methods as autonomous exploration, hand-on practice, cooperation and exchange, and self-learning by reading. Through various autonomous learning and exploration activities, students are allowed to experience the process of mathematical discovery and creation, as well as develop their innovative awareness.

(3) Mathematics teaching should reflect the process of mathematical thinking and promote the common development of teachers and students.

Mathematics teaching is the teaching of mathematical thinking activities; it is the process of interaction and mutual development between teachers and students, as well as among students. The teaching of mathematics aims to develop students' mathematical thinking ability. When students learn mathematics and use mathematical knowledge to solve problems, they constantly go through the thought processes of intuitive perception, observation and discovery, induction and analogy, spatial imagination, abstraction and generalisation, symbolic representation, performing operations and finding the solutions, data processing, deduction and proof, reflection and construction etc. Through these processes, students learn to think mathematically, improve their level of mathematical literacy and recognise the social and cultural values of mathematics.

Teachers should base their teaching on cultivating problem solving ability in students, help them to observe the world through mathematical eyes, to discover and raise questions, analyse and solve problems as well as summarise and apply findings, eventually, reflect on the learning process.

(4) Attaching importance to the integration of mathematics and information technology.

The development of information technology has had significant impact on the presentation of teaching contents, learning styles of students, teaching styles of teachers and ways of teacher-student interaction. Modern information technology is not only an important tool for exploring and presenting mathematical knowledge, but also a powerful tool that students can use to learn mathematics and solve problems. The senior secondary mathematics curriculum should advocate the realisation of the organic integration of information technology and curriculum content, e.g. incorporate algorithm into every relevant part of the mathematics curriculum. The basic principle of integration is to benefit students to learn about the nature of mathematics. The senior secondary mathematics curriculum should advocate the use of information technology to present the curriculum content that is hard to be presented in the past. On the premise of ensuring

written calculation training, teachers should use scientific calculators and various mathematics education technology platforms as much as possible to strengthen the combination of mathematics teaching and information technology, encourage students to use computers and calculators for exploration and discovery.

II. Curriculum Goals

- (1) Enable students to acquire the important mathematical knowledge, basic mathematical thought and essential application skills which are necessary for social life, practical activities and their continuous study of mathematics or other related disciplines.
- (2) Foster students' arithmetic ability, inferential ability, spatial imagination ability and data analysis ability.
- (3) Enhance students' ability to apply mathematical thinking to discover and raise questions as well as to analyse and solve problems. Foster their abilities in mathematical expression and exchange; help them to develop the ability of acquiring mathematical knowledge independently.
- (4) Develop students' awareness of mathematics application and innovative sense so that they can ponder on and judge some mathematical models embedded in the real world.
- (5) Help students to realise the scientific, application and humanistic values of mathematics; foster the habit of critical thinking in students; allow students to experience success in imaginative and creative activities; increase their interest in learning mathematics; temper their willpower to overcome difficulties; enable them to become confident and creative learners.

III. The Requirements of Basic Academic Attainments in different learning domains

Explanation of coding:

- (1) The capital English letters represent the requirements of basic academic attainments in different learning domains; A - Numbers and Algebra, B - Geometry, C - Probability and Statistics, D – Others, E - Emotions, attitudes and values.
- (2) The first number following the English letter represents the serial number of learning categories in the respective learning domain.
- (3) The second number following the English letter represents the serial number of the requirement of basic academic attainments in the respective learning category.

Learning domain A: Numbers and Algebra

Learning category A-1: Set

- A-1-1 Understand the meaning of sets and the relation of belonging between a set and its elements.
- A-1-2 Be able to use language, graphs and symbols to express a set; realise the significance and effects of using different methods of representation of a set.
- A-1-3 Know the inclusion relation between sets.
- A-1-4 Understand the meaning of set equality.
- A-1-5 Understand the concept of subsets.
- A-1-6 Understand the meaning of empty sets and universal sets.
- A-1-7 Understand the meaning of intersection of sets; be able to find the intersection of given sets.
- A-1-8 Understand the meaning of union of sets; be able to find the union of given sets.
- A-1-9 Understand the meaning of the complement of a subset in a given set; be able to find the complement of a given subset.
- A-1-10 Be able to perform mixed operations of intersection, union, and complement of sets.
- A-1-11 Be able to use Venn diagrams to illustrate set relations and operations; realise the function of intuitive graphs in understanding abstract concepts.

Learning category A-2: Number and mathematical expression

- A-2-1 Understand the concept of polynomial.
- A-2-2 Understand the synthetic division of polynomials; comprehend the remainder theorem.
- A-2-3 Understand the fundamental theorems of the polynomials listed below:
 Theorem 1: $f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$ ($a_0 \neq 0$) is the n-degree polynomial of x; if there are n different numbers, b_1, b_2, \cdots, b_n that make $f(x) = 0$, then $f(x) = a_0(x - b_1)(x - b_2) \cdots (x - b_n)$.
- Theorem 2: If the n-degree polynomial $f(x)$ is 0 for (n+1) different numbers, then $f(x) = 0$.
- A-2-4 Be able to find undetermined coefficients using the numerical value substitution method and coefficient comparison method.
- A-2-5 Understand the formulas for the sum and difference of two cubes.
- A-2-6 Realise the application of polynomial knowledge to find the root of equations.
- A-2-7 Understand the concept of rational exponents.

- A-2-8 Understand the properties of exponents and be able to perform operations with exponents.
- A-2-9 Understand the concept of general radicals.
- A-2-10 Understand the properties of radicals.
- A-2-11 Be able to perform operations with radicals, simplify fractions with irrational numbers, use formula to perform four arithmetic operations on irrational numbers and radicals as well as simplify fractions whose denominators contain $a \pm \sqrt{b}$ and $\sqrt{a} \pm \sqrt{b}$.
- A-2-12 Comprehend the concept of logarithms.
- A-2-13 Comprehend the correspondence relationship between the concept of logarithms and that of exponents and know the mutual conversion between the two.
- A-2-14 Know that there are two important logarithms: common logarithms and natural logarithms.
- A-2-15 Master the properties of logarithms and the change-of-base formula for logarithms.
- A-2-16 Be able to perform four arithmetic operations on logarithms.

Learning category A-3: Equations

- A-3-1 Understand the concept of irrational equations.
- A-3-2 Understand the solutions of irrational equations.
- A-3-3 Understand the definitions of exponential equations and logarithmic equations.
- A-3-4 Know several basic types of exponential equations and logarithmic equations as well as their solution methods.

Learning category A-4: Inequalities

- A-4-1 Understand the basic inequality $\frac{a+b}{2} \geq \sqrt{ab}$, and the algebraic proof process of $a \geq 0, b \geq 0$.
- A-4-2 Be able to apply basic inequalities to the proof and solution of other problems.
- A-4-3 Understand the concept of inequalities that contain absolute values.
- A-4-4 Be able to solve simple inequalities with absolute values.
- A-4-5 Understand the concept of quadratic inequalities in one unknown; and be able to express relevant problems using quadratic inequalities in one unknown.
- A-4-6 Realise the relations between quadratic inequalities in one unknown and the corresponding equations and functions.
- A-4-7 Be able to use the discriminant method to solve quadratic inequalities in one unknown.
- A-4-8 Be able to use factorisation to solve quadratic inequalities in one unknown.

- A-4-9 Understand the concept of higher-order inequalities in one unknown.
- A-4-10 Understand that number axis can be used to solve factorised higher-order inequalities.
- A-4-11 Comprehend the concept of linear inequalities in two unknowns.
- A-4-12 Understand the geometrical significance of linear inequalities in two unknowns and the systems of linear inequalities in two unknowns; be able to sketch the plane region that a system of linear inequalities represents.
- A-4-13 Be able to abstract some simple linear programming problems in two unknowns from real life situations.
- A-4-14 Be able to solve simple linear programming problems with two unknowns; experience the application of optimisation thinking in real life.

Learning category A-5: Function

- A-5-1 Know that function is an important mathematical model to describe the dependencies between variables.
- A-5-2 Understand the role of correspondence relation in describing the concept of functions.
- A-5-3 Understand the elements that compose a function; understand the concept of the domain and range of a function; know that the domain of a function is a prerequisite for the study and description of the properties of a function; be able to find the domain and range of some simple functions.
- A-5-4 Represent functions through the proper use of the method of images, the table method and the analytical method based on real situations.
- A-5-5 Understand simple piecewise functions and be able to use them in a simple way.
- A-5-6 Understand the concept of a monotonic function; be able to find monotone intervals of common functions.
- A-5-7 Understand the concept of even and odd functions; understand the graphical features of even and odd functions.
- A-5-8 Be able to use the respective properties to describe the graphs of functions and use the graphs of functions to understand and study function properties.
- A-5-9 Understand the concept of inverse functions.
- A-5-10 Grasp the concept of power functions and the concepts of the power functions whose exponents are $-1, -\frac{1}{2}, 2, 3$ as well as their graphs and properties.
- A-5-11 Combining with practical examples, understand the real context of exponential functions and understand that the exponential function is an important function model.
- A-5-12 Master the concept, properties and graphs of exponential functions.
- A-5-13 Through specific examples, understand intuitively the quantitative relationship described by logarithmic functions; comprehend the concept of logarithmic functions and realise that logarithmic function is an important function model.

- A-5-14 Understand that the exponential function $y = a^x$ ($a > 0$ and $a \neq 1$) is the inverse of the logarithmic function $y = \log_a x$ ($a > 0$ and $a \neq 1$), and vice versa.
- A-5-15 Comprehend the graphs and properties of logarithmic functions.

Learning category A-6: Numerical Sequence

- A-6-1 Understand the concept of numerical sequence and their expression methods (tables, graphs, and formula of general term) and understand that a numerical sequence is a special kind of function.
- A-6-2 Understand the recursion formula of a numerical sequence and be able to use the formula to find the first few terms of a numerical sequence and guess the formula of general term of certain numerical sequences.
- A-6-3 Understand the concept of arithmetic sequences and be able to determine whether or not a numerical sequence is an arithmetic sequence.
- A-6-4 Master the formula of general term of arithmetic sequences and the derivation method; experience the process of exploring the properties of arithmetic sequences.
- A-6-5 Master the formula of the sum of the first n terms and derivation method of arithmetic sequences; be able to apply the formula of general term and the sum of the first n terms' formula skillfully.
- A-6-6 Be able to solve comprehensive problems related to arithmetic sequences and establish the model of an arithmetic sequence to solve practical problems.
- A-6-7 Understand the concept of geometric sequences; be able to determine whether or not a numerical sequence is a geometric sequence.
- A-6-8 Master the formula of general term of a geometric sequence and its derivation method; experience the process of exploring the properties of a geometric sequence.
- A-6-9 Master the formula of the sum of the first n terms of geometric sequences and its derivation method; be able to use formula of general term, and the formula of the sum of the first n terms skillfully.
- A-6-10 Be able to solve comprehensive problems related to geometric sequences; and establish the model of a geometric sequence to solve practical problems.

Learning domain B: Geometry

Learning category B-1: Solid geometry

- B-1-1 Use physical models and computer software to observe a large number of spatial images. Recognise the structural features of cylinders, cones, spheres and the simple combination of these shapes and be able to describe the structural features of simple objects in real life.
- B-1-2 Know the formulas for calculating the surface area and volume of a sphere, a prism and a pyramid.

B-2 Plane analytic geometry

- B-2-1 Explore the geometric elements that determine the positions of straight lines

- in the plane Cartesian coordinate system based on specific graphs.
- B-2-2 Understand the concepts of inclination angle, slope and intercept of a straight line and be able to perform calculations.
- B-2-3 Be able to determine if two straight lines are parallel or perpendicular based on the slope; identify the slope relationship between two straight lines according to their parallelism or perpendicularity.
- B-2-4 Based on the geometric elements that determine the positions of straight lines, explore and master certain forms of straight line equations, including: the point slope form, slope-intercept form and general form and realise the relations between the slope-intercept form and linear functions.
- B-2-5 Be able to find the intersection coordinates of two straight lines using the methods for solving equations.
- B-2-6 Explore and master the point to point distance formula, the point-line distance formula and the midpoint formula; be able to find the distance between two parallel straight lines.
- B-2-7 Review the geometric elements of a circle; explore and master the standard equation and general equation of a circle in the plane Cartesian coordinate system.
- B-2-8 Be able to determine the positional relations between a straight line and a circle or between circles based on a given straight line or circle equation.
- B-2-9 Be able to find the equation of tangent to a circle according to the given slope, tangent point and a point outside the circle.
- B-2-10 Be able to solve simple problems using straight line equations and circle equations.

Learning category B-3: Conic sections and equations

- B-3-1 Understand the real context of conic sections; experience their application in real world and their role in solving practical problems.
- B-3-2 Experience the process of abstracting elliptical models from real situations and master the definition, standard equations and simple geometric properties of ellipses.
- B-3-3 Understand the definition, geometric figures and standard equations of parabolas and hyperbolas; realise their simple geometric properties.

Learning category B-4: Plane vectors

- B-4-1 Understand the actual background and basic concepts of plane vectors.
- B-4-2 Be able to distinguish vectors from vector norms and find the norm of a vector.
- B-4-3 Be able to write directed line segment notation and coordinate notation for a vector.
- B-4-4 Master the addition of vectors through real examples, and find the difference of vectors by using the inverse operation of addition.
- B-4-5 Be able to use the rules of the triangle and the parallelogram to find the sum

- of two vectors; be able to use the method of polygons and coordinates to find the sum of n vectors.
- B-4-6 In vector multiplication, be able to distinguish the difference between the product of multiplying a vector by a scalar and the inner product of vectors and explain their geometrical significance.
- B-4-7 Be able to apply the three basic operational rules of multiplying vectors by scalars to perform calculations: $m(n\vec{a}) = (mn)\vec{a}$ 、 $m\vec{a} + n\vec{a} = (m+n)\vec{a}$ 、 $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$.
- B-4-8 Be able to use the inner product formula of vectors to perform calculations, proving whether two vectors are perpendicular or parallel, and solve some geometric and algebraic problems.
- B-4-9 Be able to solve some plane geometry problems by using vectors.

Learning category B-5: Triangles

- B-5-1 Understand the concept of arbitrary angles.
- B-5-2 Understand the radian system and be able to convert angles to radians and vice versa.
- B-5-3 Master the definition of the trigonometric function for arbitrary angles; and be able to express them with the help of the directed line segments in a unit circle.
- B-5-4 Master the formulas of the basic relationships between trigonometric functions of the same angle: $\sin^2 x + \cos^2 x = 1$; $\tan x = \frac{\sin x}{\cos x}$.
- B-5-5 Understand the induction formula, and be able to apply it in a simple way.
- B-5-6 Master the sum and difference formulas for cosine and be able to apply them skillfully.
- B-5-7 Master the sum and difference formulas for sine, and be able to apply them skillfully.
- B-5-8 Understand the sum and difference formulas for tangent.
- B-5-9 Understand the double-angle formulas and be able to apply them skillfully.
- B-5-10 Through real examples, master the graphs and basic properties of sine functions.
- B-5-11 Understand the graphs and properties of cosine functions and tangent functions.
- B-5-12 Based on specific examples, master the graphs and properties of the function: $y = A\sin(\omega x + \varphi)$.
- B-5-13 Master the Law of Sines and the Law of Cosines and be able to use them to solve oblique triangles.
- B-5-14 Explore the side-angle relationships of triangles with the help of the Laws of Sines and Cosines, and determine the types of triangles.
- B-5-15 Be able to solve practical geometry problems by using the Laws of Sines and Cosines.
- B-5-16 Be able to solve some measurement problems using knowledge of triangles.

Learning domain C: Probabilities and Statistics**Learning category C-1: Probabilities**

- C-1-1 Understand the significance of space of elementary events and the significance of the addition and multiplication of events.
- C-1-2 Be able to use the learned permutation and combination knowledge to calculate the number of elementary events contained in certain random events.
- C-1-3 Through real examples, understand classical probability and its formula for calculating probability.
- C-1-4 Through real examples, understand the probability addition formulas for two mutually exclusive events.
- C-1-5 Understand the concept of conditional probability and the mutual independence of two events in specific situations.

Learning category C-2: Statistics

- C-2-1 Know how to use appropriate methods to draw samples from the population.
- C-2-2 Be able to organise sample data using appropriate methods.
- C-2-3 Through real examples, understand the significance and role of the standard deviation of sample data and know how to calculate the standard deviation of data.
- C-2-4 Estimate the numerical characteristics of a population based on the numerical characteristics of a sample.
- C-2-5 Understand the application of the numerical characteristic of samples in daily life.

Learning category C-3: Permutation, combination, and the Binomial Theorem

- C-3-1 Understand the addition and multiplication principles of counting.
- C-3-2 Be able to use the addition and multiplication principles of counting to analyse and solve some simple practical problems.
- C-3-3 Understand the concepts of permutation and the number of permutation; realise the derivation of the formula for the number of permutations.
- C-3-4 Master the formula for the number of permutations and its variants; and be able to use the number of permutations to conduct related operations.
- C-3-5 Understand some common methods for solving permutation problems with restrictions; and be able to solve some simple permutation problems.
- C-3-6 Understand the concepts of combination and combination number; know about the relationship and differences between combination and permutation.
- C-3-7 Master the combination number formula, understand and be able to apply the two properties of combination numbers.
- C-3-8 Understand some common methods for solving combination problems and be able to solve some simple combination problems.
- C-3-9 Understand the binomial theorem and its proof process.

- C-3-10 Understand the Binomial Theorem general formula and be able to use the general formula to find a specific term or the coefficient of a specific term.
- C-3-11 Understand the difference between the binomial coefficient C_n^k of a certain term in a binomial expansion and the coefficient of this term.
- C-3-12 Use the Pascal's triangle table to comprehend the symmetry, increase and decrease as well as the maximum value of binomial coefficients.

Learning domain D: Others

Learning category D-1: Derivatives

- D-1-1 Understand the actual background of concept of derivatives; know that the "instantaneous rate of change" is the derivative.
- D-1-2 Through function graphs, understand the geometrical significance of derivatives - the tangent slope; be able to use derivatives to find the equation of a tangent line and the equation of a normal line to a curve at a given point.
- D-1-3 Be able to find derivatives of simple functions using the derivative formulas of basic elementary functions and the four basic arithmetic operations of derivatives.
- D-1-4 Recognise the corresponding relationship between the monotonicity of functions and derivatives.
- D-1-5 Understand the differences between the maximum value, the minimum value and local maximum, local minimum; use derivatives to find the local maximum, local minimum and the maximum value, the minimum value for some functions and solve some simple application problems.

Learning domain E: Emotions, attitudes and values

- E-1-1 Actively participate in such mathematical activities as observation, operation, conclusion, conjecture, verification; be able to express and exchange one's thinking process.
- E-1-2 Be able to identify mathematical problems in real situations; and be able to use mathematical methods to analyse and solve problems.
- E-1-3 Be able to classify and summarise the learnt knowledge; and build up the connection between mathematical knowledge.
- E-1-4 Solve problems by creating a mathematical model, realise the application of mathematics in daily life, enhance interests in learning mathematics.
- E-1-5 In exploratory activities, be able to listen to and cooperate with others; and respect other people's point of view.
- E-1-6 Be able to overcome the difficulties encountered in mathematical problem solving; improve self-confidence in learning mathematics; cultivate the habit of thinking carefully as well as the down-to-earth attitude.