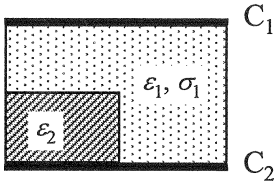
Pan Pearl River Delta Physics Olympiad 2009
2009年泛珠三角及中華名校物理奧林匹克邀請賽
Part-1 (Total 5 Problems) 卷-1（共5題）
(9:00 am – 12:00 pm, 02-04-2009)

Math hints 數學提示: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$; $\int \frac{dx}{x} = \ln x + C$.

Q.1 (8 points) 題1（8分）
Two large parallel square conductor plates C_1 and C_2 of side length a are separated by two types of media. The cross section is shown in the figure. The distance between the plates is d. Medium-2 is a rectangular insulator slab with dielectric constant ε_2, thickness $d/2$, and side lengths a and $a/2$, respectively. The rest of the space between the plates is filled with medium-1 with conductivity σ_1 and dielectric constant ε_1. Find the resistance and capacitance between the two plates.



兩塊邊長爲 a 的正方形平行導電板 C_1 和 C_2 之間夾著兩種介質，其截面如圖所示。導電板間的距離爲 d。介質-2 爲一厚度爲 $d/2$，邊長分別爲 a 和 $a/2$ 的絕緣長方形板，其介電常數爲 ε_2。導電板之間其餘的空間充滿了介質-1，其介電常數爲 ε_1，電導率爲 σ_1。求兩導電板之間的電阻和電容。

Q.2 (10 points) 題2（10分）
A galaxy is made of many stars revolving around its center on circular orbits. One way scientists study galaxies is to measure the relation between the revolving speed v of a star in the galaxy and its distance r from the center. Expressing the relation as $v \propto r^n$, scientists pay special attention to the exponential power n.

星系由很多繞中心作圓形軌道運行的恒星組成。科學家研究星系的一個方法是測量恒星在星系中的運行速度 v 和離星系中心的距離 r。用 $v \propto r^n$ 這樣的關係來表達，科學家們特別關心指數 n。

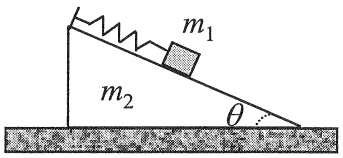
(a) If the gravity force on the star under investigation is mainly from a massive black hole at the galaxy center, derive the value of n. (4 points)
若作用于恒星的引力主要來自星系中心的巨型黑洞，求 n 的值。（4分）

(b) Suppose there is no black hole at the center and the stars in the galaxy are evenly distributed such that the mass of the galaxy is evenly distributed in a flat disk. Consider a star in the disk at distance r from the center. The total gravity force on the star from the stars enclosed by its circular orbit can be treated as if they are concentrated at the center. The total gravity force on the star from the stars outside the orbit is zero. (This is like the case of a uniform density sphere.) Derive the value of n. (4 points)
若星系中心無黑洞，星系的恒星分布均勻，整個星系可當作是一個質量均勻分布的扁圓盤。對于一個離中心爲 r 的恒星來講，來自軌道內的恒星的引力與把它們集中在星系中心的一樣，來自軌道外的恒星的引力合力爲零。（類似于均勻質量球的情形）求 n 的值。（4分）

(c) What scientists found from real galaxies is that for most of them, n is larger than 1. Give your brief explanation to this scientific puzzle. (2 points)
科學家們發現，很多實際星系的 n >1。請簡單解釋。（2分）

Q.3 (10 points) 題 3（10分）

As shown, a block of mass m_1 is connected to a spring of force constant k on the smooth slope (inclination angle θ) of a wedge of mass m_2 placed on a smooth floor. Given a small disturbance to the block and the system starts to oscillate. During the oscillation motion the block keeps in touch with the slope, and the wedge maintains contact with the floor. Find the oscillation frequency, and check your answer for two special cases of $\theta = 0$ and $\theta = 90°$.



如圖所示，一放在光滑地板上質量爲 m_2 的大物塊的光滑斜面上（傾角爲 θ）有一力常數爲 k 的彈簧，彈簧上系有質量爲 m_1 的小物塊。給小物塊一輕微擾動後，系統開始振蕩。在振蕩期間小物塊始終與斜面保持接觸，大物塊始終與地板保持接觸。求振蕩頻率，并用 $\theta = 0$ 和 $\theta = 90°$ 這兩個特殊情形檢驗你的答案。

Q.4 (10 points) 題 4（10分）

An electromagnetic (EM) wave is propagating along the $\vec{z}_0$ direction in a non-magnetic conducting medium with conductivity σ and dielectric constant ε. Both constants are real numbers. Its electric field is $\vec{E}(z,t) = E_0 \vec{x}_0 e^{i(\tilde{k}z - \omega t)}$, where E_0 and ω are real constants while $\tilde{k} = \frac{\omega}{c}\sqrt{\varepsilon} + i\frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon\varepsilon_0}}$.

一電磁波在非磁性導電介質中沿 $\vec{z}_0$ 方向傳播。介質的導電率爲實數 σ，介電常數爲實數 ε。電磁波的電場爲 $\vec{E}(z,t) = E_0 \vec{x}_0 e^{i(\tilde{k}z - \omega t)}$，其中 $\tilde{k} = \frac{\omega}{c}\sqrt{\varepsilon} + i\frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon\varepsilon_0}}$，$E_0$ 和 ω 爲實數。

(a) Find the magnetic field of the EM wave. (2 points)
求電磁波的磁場。（2分）

(b) Find the time-averaged (over one period) Poynting's vector $< \vec{S} > = \frac{1}{\mu_0} < (\vec{E} \times \vec{B}) >$. (2 points)
求經過一個周期的時間平均的 Poynting 矢量 $< \vec{S} > = \frac{1}{\mu_0} < (\vec{E} \times \vec{B}) >$。（2分）

(c) The quantity $q = -\frac{d < S >}{dz}$ describes the loss of EM wave energy to a unit volume of the medium. Calculate q. (2 points)
$q = -\frac{d < S >}{dz}$ 代表電磁波輸送給單位體積的介質的能量。求 q。（2分）

(d) Find the power consumed by a unit volume of the medium by directly calculating the Joule Heat averaged over one period of the wave $< \vec{J} \cdot \vec{E} >$, where $\vec{J}$ is the current density. (2 points)
求單位體積的介質以焦耳熱形式經過一個周期的時間平均的能量消耗 $< \vec{J} \cdot \vec{E} >$，其中 $\vec{J}$ 爲電流密度。（2分）

(e) Compare the answers in (c) and (d) and explain why they are equal or not equal. (2 points)
比較(c)和(d)的答案，解釋它們相等或不相等的原因。（2分）

Q.5 (12 points) 題5（12分）

Consider a **non-ideal** gas with internal energy U given by $U = 3PV$, where P and V are the pressure and volume of the gas, respectively.
一非理想氣體的內能爲 $U = 3PV$，其中 P 爲氣體的壓强，V 爲氣體的體積。

(a) Find the relation between P and V in an adiabatic process. (2 points)
求絕熱過程 P 與 V 的關係式。（2分）

Experiments show that the temperature T of the gas depends only on P, irrespective of its volume. The scale of the temperature can be fixed by setting $T = 1$ when $P = 1$. Consider the following Carnot Process
實驗證明該氣體的溫度只與壓强有關，而與體積無關。溫度的標度可用當 $P = 1$ 時 $T = 1$ 來定。考慮以下的卡諾過程

$$(P_1,V_1) \overset{\text{Isothermal}}{\rightarrow} (P_1,V_2) \overset{\text{Adiabatic}}{\rightarrow} (P_2,V_3) \overset{\text{Isothermal}}{\rightarrow} (P_2,V_4) \overset{\text{Adiabatic}}{\rightarrow} (P_1,V_1)$$

（Isothermal = 等溫；Adiabatic = 絕熱）

(b) Calculate the heat Q_1 absorbed by the gas in the first isothermal process. (1 point)
求第一個等溫過程氣體吸收的熱量 Q_1。（1分）

(c) Calculate the heat Q_2 absorbed by the gas in the second isothermal process. (1 point)
求第二個等溫過程氣體吸收的熱量 Q_2。（1分）

(d) The temperature T can be defined as $\frac{T_1}{T_2} = -\frac{Q_1}{Q_2}$. Find the $P \sim T$ relation. (4 points)

溫度可定義爲 $\frac{T_1}{T_2} = -\frac{Q_1}{Q_2}$。求 P 與 T 的關係式。（4分）

(e) Find the heat capacity at constant volume of the gas. (2 points)
求氣體的恒定體積熱容量。（2分）

(f) Find the entropy S of the gas in terms of P and V, given that $S = 0$ when $T = 0$. (2 points)
已知氣體的熵 S 在溫度 $T = 0$ 時爲零。求氣體的熵，并以 P 和 V 表達。（2分）

THE END 完