Pan Pearl River Delta Physics Olympiad 2009
2009 年泛珠三角及中華名校物理奧林匹克邀請賽
Part-2 (Total 3 Problems) 卷-2（共 3 題）
(2:30 pm – 5:30 pm, 02-04-2009)

Math hints 數學提示: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$；$\int \frac{dx}{x} = \ln x + C$.

Q1 Solar System in the Early Days (16 points) 題 1 早期的太陽系（16 分）

In the early days of our solar system when the planets were not yet formed, the sun was surrounded by a big ball of gas at rest. The mass of the sun is M_s. Assume for simplicity that the gas is ideal and is made of single type of molecules of molecular mass m, and the total mass of the gas is much less than M_s, you are to find the distribution of the gas at a distance r (>> diameter of the sun) from the sun.

早期的太陽系行星還沒有形成時，太陽被一大團氣體包圍著。已知太陽的質量爲 M_s。爲簡單起見設氣體是理想的，并由單種分子質量爲 m 的分子組成。氣體的總質量遠小于 M_s。你需要求出離太陽距離 r（>> 太陽半徑）處氣體的分布。

(a) Suppose the gas is all at the same temperature T_0. The mass density distribution of the gas can be expressed as $\rho = \rho_0 e^{\alpha/r}$. Find α. (3 points)

假設氣體的溫度爲 T_0 并且是均勻的，氣體的質量密度分布可表達成 $\rho = \rho_0 e^{\alpha/r}$，求α。（3 分）

(b) There is a major flaw in the mass density expression in (a). Point out the flaw. (2 points)
上述氣體的質量分布表達式有一重大缺陷。指出缺陷。（2 分）

(c) In an improved model, suppose the sun emits J_0 amount of thermal energy per second, and there is no energy loss when the thermal energy flows from the sun out to the gas via thermal conduction, find the energy current density (energy through a unit area per second) at a distance r from the sun. (3 points)

在改良的模型裏，假設太陽每秒釋放出的熱能爲 J_0，當熱能以熱傳導方式從太陽傳給氣體時無損失，求離太陽距離 r 處熱能量流强度（每秒穿過單位面積的能量）。（3 分）

(d) The thermal energy current density $I(r)$ in (c) is proportional to the temperature gradient. That is, $I(r) = -\sigma \frac{dT}{dr}$, where σ is a positive constant called heat conductivity. The minus sign comes from the fact that heat always flows from high T region to low T region. Find the temperature at distance r from the sun. (3 points)

上述熱能量流强度 $I(r)$ 與溫度梯度成正比，既 $I(r) = -\sigma \frac{dT}{dr}$，其中 σ 爲熱導率，是正數，式中的負號是因爲熱流總是從高溫區域流向低溫區域的。求離太陽距離 r 處溫度。（3 分）

(e) The pressure can now be expressed as $P = P_0 (r/r_0)^{-\beta}$. Find β and the mass density distribution. (3 points)

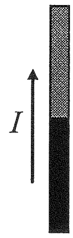
氣體的壓强可表述爲 $P = P_0 (r/r_0)^{-\beta}$，求 β 以及質量密度分布。（3 分）

(f) From (d) one can see that beyond certain distance r_0 from the sun the temperature is below the ice temperature. Estimate r_0 in terms of the radius of a planet orbit of the present solar system. (2 points)

由(d)可知超過某距離 r_0 氣體的溫度會低于水的冰點。r_0 大概與現有的哪個行星的軌道半徑相近？（2 分）

Q2 Spin Torque (16 points) 題 2 自旋力矩（16 分）

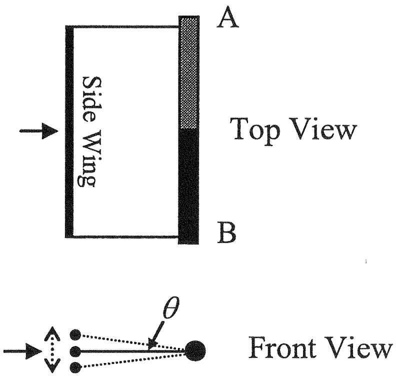
An electron has permanent angular momentum called **spin** ($\vec{S}$) that is fixed in amplitude $h/4\pi$, where h is the Planck Constant, while its direction can be changed by external interactions. Consider a thin wire with electric current I, the lower half of which is made of a magnetized conductor and the upper half non-magnetic conductor. Due to quantum mechanics, the electron spins can only point to one of the two directions, namely in the electric current direction or opposite to it. In the magnetized section of the wire there are α (> 50%) portion of the electrons with spins in the current direction, while in the non-magnetic section there are equal numbers of electrons with spins in the two directions. The extra number of electrons with spins in the current direction in the magnetic section will instantly flip to the opposite direction once they cross the boundary and enter the non-magnetic section. As a result, a net torque is exerted on the wire.



電子具有固有的角動量$\vec{S}$，叫做自旋，其方向可在與外界的相互作用下改變，其大小爲固定的$h/4\pi$，其中 h 爲普朗克常數。一導綫載有電流 I，導綫的下半部分由磁化了的導體組成，上半部分由非磁性導體組成。由量子力學得知，電子的自旋只能與電流同方向，或與電流反方向。磁化導體內有α（> 50%）部分的電子的自旋與電流同方向。非磁性導體內自旋與電流同方向和反方向的電子具有相同數量。磁化導體內多出的自旋與電流同方向的電子流進非磁性導體後會立刻改變它們的方向，從而產生力矩作用在導綫上。

(a) Find the torque in terms of h, I, α, and electron charge e. (4 points)
以 h, I, α, 和電子電荷 e 來表達力矩。（4 分）

(b) To measure the torque, a conducting side wing is attached rigidly to the wire but is electrically insulated from it, as shown in the figure. The wire is suspended between two fixed points A and B. Under constant current I_0 the spin torque will twist the entire wire along the AB axis up to a small angle θ_0, until it is balanced by the elastic restoring torque $\tau_e = -\kappa\theta_0$, where κ is a positive constant, at the two fixed points. Given that the moment of inertia of the entire structure with respect to the AB axis is J, and there is also a friction torque of $-\eta\dfrac{d\theta}{dt}$, where η is a positive constant, find the angle θ as a function of time t after the current is suddenly turned off at $t = 0$. (6 points)



爲了測量力矩，我們在導綫上裝上一導電邊翼(side wing) ，如俯視圖(top view)和正視圖(front view)所示。邊翼與導綫的連接是剛性的，但相互間不導電。導綫的兩端固定在 A、B 兩點，中間懸空。有恒定電流 I_0 時，自旋力矩將導綫以 AB 爲軸扭轉一小角θ_0，直到被 A、B 點的彈性力矩$\tau_e = -\kappa\theta_0$ 抵消爲止，其中κ是正常數。已知整個結構以 AB 爲軸的轉動慣量爲 J，摩擦力矩爲$-\eta\dfrac{d\theta}{dt}$，其中η爲正常數。在時間 $t = 0$ 時電流被突然切斷，求θ之後的演化。（6 分）

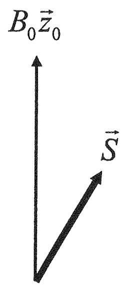
(c) When an alternating electric current $I = I_0 e^{i\omega t}$ is applied, the wire will twist back and forth. The side wing of length L at a distance d from the wire follows the twisting motion of the wire. A magnetic field B is applied perpendicular to the side wing, as shown by the solid arrow in the figure. The field will not change the spins. Find the electromotive potential between the two ends of the side wing. (6 points)

將導綫通上交流電$I = I_0 e^{i\omega t}$，則導綫會來回扭動，帶動邊翼上下擺動。已知邊翼的長度爲L，離導綫的距離爲d，外加恒穩磁場場强爲B，方向與邊翼垂直，如圖中實綫箭頭所示。該磁場不會改變電子的自旋。求邊翼兩端之間的電動勢。（6分）

Q3 Nuclear Magnetic Resonance (18 points) 題3 核磁共振 (18分)

Like the electrons, most nuclei have permanent angular momentum called **spins** ($\vec{S}$) that are fixed in amplitude while their directions can be changed by an external torque. Associated with the spins are the magnetic moments that make them as individual magnetic dipoles $\vec{m} = \mu\vec{S}$, where μ is a constant. Here we will study the spin dynamics using classical mechanics.

大多數原子核象電子那樣具有叫作自旋的固有角動量$\vec{S}$，其大小固定，方向隨外力矩的作用而改變。由自旋產生的磁偶極子爲$\vec{m} = \mu\vec{S}$，其中μ爲常數。下面我們用經典力學研究自旋的運動。



(a) A nucleus with spin $\vec{S}$ is placed in a static magnetic field $B_0\vec{z}_0$ in a laboratory, as shown in the figure. Find the motion of the spin of the nucleus as seen in the laboratory reference frame. (4 points)
如圖，一自旋爲$\vec{S}$的原子核處于實驗室的靜磁場$B_0\vec{z}_0$中。求在實驗室參照系觀察到的自旋的運動。（4分）

(b) In a rotating reference frame with angular velocity $\vec{\omega}_0$ the nucleus spin appears stationary. Find such reference frame. (2 points)
在一個以角速度$\vec{\omega}_0$旋轉的參照系觀察到的自旋是固定不動的。找出該參照系。（2分）

(c) The correct answer to (b) indicates that in a rotating reference frame with angular velocity $\vec{\omega}$ there is an additional magnetic field $\vec{B}_\omega$ seen by the spin. Find the additional magnetic field. (2 points)
上述(b)的正確答案說明，在一個以角速度$\vec{\omega}$旋轉的參照系裏，自旋會感受到一個額外的磁場$\vec{B}_\omega$。求額外磁場。（2分）

(d) Back in the laboratory reference frame a second magnetic field is applied. The second field with strength B_1 is always perpendicular to $\vec{z}_0$ but its direction rotates in the XY plane at constant angular speed ω_1. Switching to the reference frame that rotates with the second magnetic field, find the total magnetic field seen by the nucleus. (3 points)
在實驗室參照系，我們加上另一個磁場。該磁場強度爲B_1，其方向與$\vec{z}_0$垂直，并在XY平面內以均勻角速度ω_1旋轉。在隨著該磁場旋轉的轉動參照系裏，求自旋感受到的總磁場。（3分）

(e) When $\omega_0 = \omega_1$, find the minimum time it takes to flip the spin from $+\vec{z}_0$ to $-\vec{z}_0$. (This situation is called the Nuclear Magnetic Resonance) (4 points)
當$\omega_0 = \omega_1$時，求將自旋從$+\vec{z}_0$方向轉到$-\vec{z}_0$方向所需的最短時間。（這情形稱爲核磁共振）（4分）

(f) When $\omega_0 \neq \omega_1$, find the motion of the spin observed in the rotating frame. (3 points)
當$\omega_0 \neq \omega_1$時，求在轉動參照系裏觀察到的自旋運動。（3分）

THE END 完